V

ON THE ACTION OF A RECTILINEAR ELECTRIC OSCILLATION UPON A NEIGHBOURING CIRCUIT

(*Wiedemann's Annalen*, **34**, p. 155, 1888.)

IN an earlier paper[1] I have shown how we may excite in a rectilinear unclosed conductor the fundamental electric oscillation which is proper to this conductor. I have also shown that such an oscillation exerts a very powerful inductive effect upon a nearly closed circuit in its neighbourhood, provided that the period of oscillation of the latter is the same as that of the primary oscillation. As I intended to make use of these effects in further researches, I examined the phenomenon in all the various positions which the secondary circuit could occupy with reference to the inducing current. The total inductive action of a current-element upon a closed circuit can be completely calculated by the ordinary methods of electromagnetics. Now since our secondary circuit is closed, with the exception of an exceedingly short spark-gap, I supposed that this total action would suffice to explain the new phenomena; but I found that in this I was mistaken. In order to arrive at a proper understanding of the experimental results (which are not quite simple), it is necessary to regard the secondary circuit also as being in every respect unclosed. Hence it is not sufficient to pay attention to the integral force of induction; we must take into consideration the distribution of the electromagnetic force along the various parts of the circuit: nor must the electrostatic force which proceeds from the charged ends of the oscillator be neglected. The reason of this is the rapidity with which

[1] See II., p. 29.

the forces in these experiments alter their sign. A slowly alternating electrostatic force would excite no sparks in our secondary conductor, even if its intensity were very great, since the free electricity of the conductor could distribute itself, and would distribute itself, in such a way as to neutralise the effect of the external force; but in our experiments the direction of the force alters so rapidly that the electricity has no time to distribute itself in this way.

For the sake of convenience I will first sketch the theory and then describe the phenomena in connection with it. It would indeed be more logical to adopt the opposite course; for the facts here communicated are true independently of the theory, and the theory here developed depends for its support more upon the facts than upon the explanations which accompany it.

The Apparatus

Before we proceed to develop the theory, we may briefly describe the apparatus with which the experiments were carried out, and to which the theory more especially relates. The primary conductor consisted of a straight copper wire 5 mm. in diameter, to the ends of which were attached spheres 30 cm. in diameter made of sheet-zinc. The centres of these latter were 1 metre apart. The wire was interrupted in the middle by a spark-gap $^3/_4$ cm. long; in this oscillations were excited by means of the most powerful discharges which could be obtained from a large induction-coil. The direction of the wire was horizontal, and the experiments were carried out only in the neighbourhood of the horizontal plane passing through the wire. This, however, in no way restricts the general nature of the experiments, for the results must be the same in any meridional plane through the wire. The secondary circuit, made of wire 2 mm. thick, had the form of a circle of 35 cm. radius which was closed with the exception of a short spark-gap (adjustable by means of a micrometer-screw). The change from the form used in the earlier experiments to the circular form was made for the following reason. Even the first experiments had shown that the spark-length was different at different points of the secondary conductor, even when the position of the conductor as a whole was not altered. Now the choice of the circular form made it easily possible to bring the spark-gap to any desired position. This was most conveniently done by mounting the circle so that it could be rotated about an axis passing through its centre, and perpendicular to its plane. This axis was mounted upon various wooden stands in whatever way proved from time to time most convenient for the experiments.

With the dimensions thus chosen, the secondary circuit was very nearly in resonance with the primary. It was tuned more exactly by soldering on small pieces of sheet-metal to the poles so as to increase the capacity, and increasing or diminishing the size of these until a maximum spark-length was attained.

Analysis of the Forces acting on the Secondary Circuit

We shall assume that the electric force at every point varies as a simple periodic function of the time, changing its sense without changing its direction; we shall further assume that this variation has the same phase at all points. This is true at any rate in the neighbourhood of the primary conductor; and for the present we shall restrict our attention to points which lie near it. Any point on the secondary circuit is determined by its distance s measured from the spark-gap along the circle. We denote by Σ the component of the electric force which acts at any moment at the point s in the direction of the element ds of the circle. Then Σ is a function of s which, after passing round the whole circumference S, returns to its original value. Σ can therefore be developed in circular functions, beginning thus—

$$\Sigma = \mathrm{A} + \mathrm{B} \cos 2\pi s/\mathrm{S} + \ \ldots \ + \mathrm{B}' \sin 2\pi s/\mathrm{S} + \ \ldots$$

We shall neglect the higher terms. The effect of this will be that our results will only be approximately correct; in especial, weak sparks will be found to occur at places where our calculations indicate that the sparking should disappear. But for the present our experiments are not sufficiently accurate to justify us in paying any attention to these higher terms. Let us therefore consider more closely the terms which have been referred to.

In the first place, the force A acts in the same sense, and is of the same magnitude at all parts of the circle. A is independent of the electrostatic force; for the integral of the latter, taken all round the circuit, is zero. A corresponds to the total induced electromotive force. We know that this is measured by the change, per unit of time, in the number of magnetic lines of force which pass through the circle. If we regard the magnetic field in which the circle lies as being homogeneous, A will be proportional to the component of the magnetic force perpendicular to the plane of the secondary circuit. A will therefore vanish when the direction of the magnetic force lies in the plane of the secondary circuit. The force A gives rise to an oscillation the intensity of which is independent of the position of the spark-gap in the circle; we shall denote by a the spark-length which corresponds to this oscillation.

Turning now to the two other terms, we note, in the first place, that the force $B' \sin 2\pi s/S$ is not in a position to excite the fundamental oscillation of our circle. For it is completely symmetrical on both sides of the spark-gap; it acts in the same sense upon both poles, and therefore cannot produce any difference between them. The force $B \cos 2\pi s/S$ behaves otherwise. If we start from the spark-gap and divide the circle into four equal parts, we find that this force acts in the same direction in the two parts which lie opposite the spark-gap, and that here it powerfully excites the fundamental oscillation. It is true that B acts in an opposite direction in those parts which lie nearest the spark-gap; but the latter parts cannot here exert as powerful an effect. For since the current at the open ends of the circle must always be zero, the electricity cannot move with the same freedom near these ends as in the middle of the circle. To elucidate the meaning of this somewhat brief statement we may take as an illustration a string stretched between two fixed points. If the middle and outer parts of the string are acted upon by forces in opposite directions, the string as a whole will move as if acted upon by the former set of forces, and the fundamental note of the string will be produced if the alternations of these forces synchronise with this note. Thus the term $B \cos 2\pi s/S$ will excite the fundamental oscillation of our circle, and the direction of the oscillation will be the same as if the force in the parts opposite the spark-gap were alone effective. Further, the intensity of the oscillation will be proportional to the quantity B. To find out what this quantity means, let us assume that the electric field in which the circle lies is approximately homogeneous. Let E denote the total electric force acting in this field, ω the angle which its direction makes with the plane of the secondary circle, and θ the angle which the projection of the force upon this plane makes with the straight line drawn from the centre to the spark-gap. Then $\Sigma = E \cos \omega \sin (2\pi s/S - \theta)$ approximately,[1] and therefore $B = -E \cos \omega \sin \theta$. Hence the value of B depends directly upon the total force; electrostatic as well as electromagnetic causes contribute towards it. B becomes zero when $\omega = 90°$, *i.e.* when the total force is perpendicular to the plane of the circle; and in this case it will be zero for all positions of the spark-gap in the circle. But B also becomes zero when $\theta = 0$, *i.e.* when the projection of the total force upon the plane of the circle coincides with the line drawn from the centre to the spark-gap. If in any given position of the circle we suppose the spark-gap to move round it, the angle θ alters, and corresponding alterations are produced in the value of B, in the intensity of the oscillation, and in the spark-length. Thus the spark-length, which corresponds to the second term of our series, can be approximately represented by the expression $\beta \sin \theta$.

The two terms which produce respectively the spark-lengths α and $\beta \sin \theta$ have always the same phase. Hence the induced oscillations have also the same phase, and their amplitudes have to be added together. Now inasmuch as the spark-lengths are approximately proportional to the total amplitudes, it follows that the spark-lengths have also to be added together. If in any given position of the circle we suppose the spark-gap to move round it, the spark-length must accordingly be represented by an expression of the form $\alpha + \beta \sin \theta$. Equal absolute values of this expression indicate equal spark-lengths, whatever the sign may be; for there is nothing in the spark-length which corresponds to direction of

[1] If the field is really homogeneous, then $A = 0$; and A will therefore be small when the field is approximately homogeneous. Nevertheless the force A may give rise to an oscillation of the same order of magnitude as that produced by the force $B \cos 2\pi s/S$.

oscillation. The absolute values of a and β could only be determined by a much more detailed development of the theory; but we have indicated the conditions upon which they depend, and this will be enough to enable us to understand the phenomena.

The Plane of the Secondary Circuit is Vertical

Let us now place our circle anywhere in the neighbourhood of the primary conductor, with its plane vertical and its centre in the horizontal plane which passes through the primary conductor. As long as the spark-gap lies in the horizontal plane, either on the one side or the other, we observe no sparks; but in other positions of the spark-gap we perceive sparks of greater or less length. The disappearance of the sparks occurs at two diametrically opposite points; it follows that the a of our formula is here always zero, and that θ becomes zero when the spark-gap lies in the horizontal plane. From this we draw the following conclusions:—In the first place, that the lines of magnetic force in the horizontal plane are everywhere vertical, and therefore form circles around the primary oscillation, as indeed is required by theory. Secondly, that at all points of the horizontal plane the lines of electric force lie in this plane itself, and therefore, that everywhere in space they lie in planes passing through the primary oscillation—which is also required by theory. If while the circle is in any one of the positions here considered, we turn it about its axis so as to remove the spark-gap out of the horizontal plane, the spark-length increases until the sparks arrive at the top or the bottom of the circle, in which positions they attain a length of 2-3 mm. It can be proved in various ways that the sparks thus produced correspond, as our theory requires, to the fundamental oscillation of our circle, and not, as might be suspected, to the first overtone. By making small alterations in the circle, for example, we can show that the oscillation which produces these sparks is in resonance with the primary oscillation; and this would not hold for the overtones. Again, the sparks disappear when the circle is cut at the points where it intersects the horizontal plane, although these points are nodes with respect to the first overtone.

If we now maintain the spark-gap at the highest point and turn the circle round about a vertical axis passing through its centre and the spark-gap, we find that during a complete revolution the sparks twice reach a maximum length and twice become zero or else very nearly disappear. Clearly the maximum positions are those in which the direction of the electric force lies in the plane of the circle ($\omega = 0$); whereas the minimum positions are those in which the direction of the electric force is perpendicular to this plane ($\omega = 90°$). Hence we now have a means of finding out the direction of the electric force at any point. I have investigated at a number of points the positions in which the sparks either become very short or completely disappear. In the lower part of Fig. 22

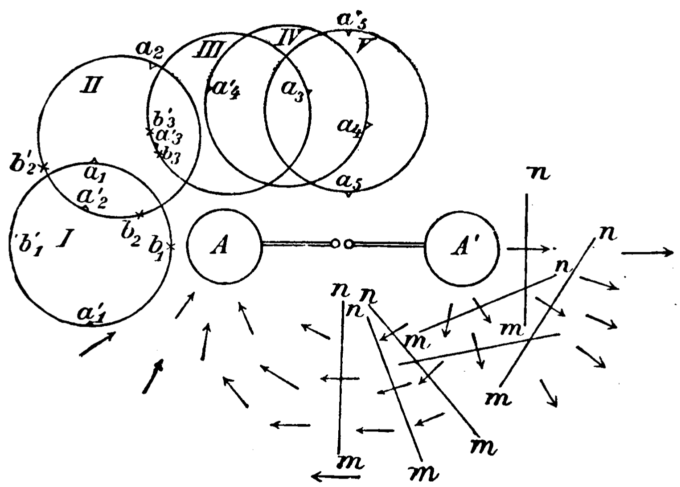

Fig. 22.

these are shown as taken directly from the experimental results. *A A′* is the primary conductor. The straight lines *m n* are the projections of the secondary conductor on the horizontal plane; but all the observed positions are not shown in the illustration. The short lines, normal to the lines *m n*, indicate the direction of the force. Since this force nowhere becomes zero, as we pass from the sphere *A* to the sphere *A′*, it does not change its sign. Hence we may furnish these normals with arrow-heads, as has been done in the figure. With regard to this figure we remark :—

1. The distribution of the total force in the neighbourhood of the rectilinear oscillation is very similar to the distribution of the electrostatic force which proceeds from the ends of the

oscillation. Near the centre of the oscillation in especial the direction of the total force coincides with that of the electrostatic force; the opposing electromagnetic force must therefore be overpowered. Theory also indicates that in this neighbourhood the force of electromagnetic induction should be weaker than the electrostatic force.

2. Still we can quite easily recognise the effect of the electromagnetic induction. For the lines of force appear to a certain extent to be pushed away from the axis of the oscillation; in going from A to A' they make a wider circuit than they would if the electrostatic force alone were in operation.[1] The explanation of this is that the force of induction weakens the components of the electrostatic force which are parallel to the primary conductor, whereas they are without influence upon the components which are perpendicular to the primary conductor.

The Plane of the Secondary Circle is Horizontal

We shall explain by reference to the upper half of Fig. 22 the phenomena which are observed when the plane of the secondary circle is horizontal. First suppose the circle to be brought into position *I*, in which its centre lies on the prolongation of the primary oscillation. After what has been already stated, we may at once conclude from purely geometrical considerations that the sparks will disappear when the spark-gap is at the points b_1 and b'_1; and also that maximum and equal spark-lengths should be observed at the points a_1 and a'_1.[1] In my experiments the lengths of these sparks were 2·5 mm.

Now let us shift the circle sideways into the position *II*. Here lines of magnetic force pass through the circle. The integral of the force of induction taken round the circle does not vanish; a is not zero. We may therefore expect to find that our expression $a + \beta \sin \theta$, in which the value of a at first is small, will have (since we pay no regard to sign) two maxima of unequal value, viz. $\beta + a$ and $\beta - a$. These will occur when $\theta = 90^\circ$, and the line joining them will be perpendicular to the direction of the electric force. These two maxima must be separated by two points at which no sparking

[1] The original drawing showed this more plainly than the reduced copy does.

occurs, and these points should lie near the smaller maximum. This agrees with the experimental results. For in the points b_2 and b'_2 we again find our null-points which have been drawn closer together; between these at a_2 and a'_2 are maximum spark-lengths, and that at a_2 is found to be 3·5 mm., while that at a'_2 is 2 mm. The line a_2 a'_2 is very nearly perpendicular to the direction of the electric force. In order to complete our explanation we have yet to show that a_2 must correspond to the sum, and a'_2 to the difference of the actions. Let us consider the case in which the spark-gap lies at a_2. While the sphere A is positively charged, the total electric force in those parts of the circle which lie opposite to a_2 urges positive electricity in a direction away from A; it tends to move positive electricity in a circular direction, which in the case of our illustration would be the direction of the hands of a clock. Between the spheres A and A' the electrostatic force at the same time is directed from A towards A'; the force of induction which is always opposed to it is therefore, in the neighbourhood of the conductor, directed towards A, and everywhere in space is parallel to this direction. Now since this force in our circle acts more strongly in the neighbourhood of the primary oscillation than it does at a distance from the latter, it follows that this force also tends to set positive electricity in motion in a circular direction corresponding to that of the hands of a clock. Hence at a_2 both causes act in the same sense and so strengthen each other. Similarly it can be shown that at a'_2 they act in opposite senses and weaken each other. Thus the phenomenon is completely explained.

Now suppose our circle to be moved nearer the centre of the primary oscillation to *III*. Here the two points at which the spark is extinguished coincide into one. One maximum disappears; and opposite to a very extended tract of extinction a'_3 lies the second maximum with a spark-length of 4 mm. Here evidently $\alpha = \beta$, and the spark-length is represented by the formula $\alpha(1 + \sin\theta)$. The line a_3 a'_3 is again perpendicular to the direction of the electric force. If we move the circle still nearer to the centre of the primary oscillation, α becomes greater than β. The expression $\alpha + \beta \sin\theta$ can no longer be zero for any value of θ, but it oscillates between a maximum value $\alpha + \beta$ and a minimum value $\alpha - \beta$.

Experiment also shows that in the positions under consideration there are no longer any points at which the sparks are extinguished; there are only maxima and minima. In position *IV* we have at a_4 a spark-length of 5·5 mm., and at a'_4 a length of 1·5 mm. In position *V* we have at a_5 a spark-length of 6 mm., at a'_5 the spark-length is 2·5 mm.,[1] and at intermediate points we have intermediate values. In passing over from position *III* to position *V*, the join $a\,a'$ turns sharply from a direction parallel to the primary current into a direction perpendicular to it; it therefore always remains approximately perpendicular to the direction of the electric force.

In the last-mentioned positions the sparks are mainly due to electromagnetic induction. Hence, in my first paper, I made no error in speaking of the phenomena in these positions as being electromagnetic effects. Nevertheless, the production of sparks even in these positions is completely independent of electrostatic causes only when we bring the spark-gap into the mean position between maximum and minimum, in which particular position $\beta \sin \theta$ becomes zero.

The Remaining Positions of the Secondary Circle

The positions which as yet have not been discussed, and in which the secondary circle is inclined to the horizontal plane, can be regarded as intermediate states between those which have already been described. In all such cases I have found the theory confirmed and have noticed no phenomenon which did not fit in with it. Let us consider one case only. Suppose the circle in the first place to lie in the horizontal plane and in position *V*, with the spark-gap turned towards the primary oscillation at a_5. Now let the circle be turned about a horizontal axis passing through its centre parallel to the primary oscillation, in such a way that the spark-gap rises. While the circle turns, the electric force is always at right angles to the straight line drawn from its centre to the spark-gap; thus θ is always equal to 90°. The value of β is approximately constant in all positions. But a varies approximately as the

[1] In these positions the secondary spark must, in order to avoid disturbing causes, be protected from the light of the primary spark.

cosine of the angle ϕ between the plane of the circle and the horizontal plane, since a is proportional to the number of lines of magnetic force cut by the circle. Thus if a_0 denote the value of a in the initial position, the value of a in any other position is $a_0 \cos \phi$, and it may therefore be expected that the relation between the spark-length and the angle ϕ may be given by the expression $a_0 \cos \phi + \beta$, where $a_0 > \beta$. Experiment confirms this. For as we raise the spark-gap the sparking distance steadily decreases from its initial value of 6 mm. and acquires at the highest point in its circuit a length of 2 mm. It then sinks farther in the second quadrant almost to zero, increases again to the smaller maximum of 2·5 mm., which occurs in the horizontal plane, again decreases, and after passing through the same stages in the reverse order it returns to its original value.

Let us suppose that in the course of the movements above described we hold the circle in the position in which the spark-gap is at its highest point. If now we raise the circle vertically as a whole, the sparks become weaker and ultimately they almost disappear; if we lower the circle vertically the sparking becomes more vigorous. But if under similar circumstances the spark-gap is at its lowest point, the effects are reversed. These results may be deduced by purely geometrical reasoning from what has been already stated.

The Forces at Greater Distances

We have already mentioned a method of ascertaining experimentally the direction of the total electric force at any point. There was no difficulty in extending the application of this method to greater distances, and there was all the more reason for making the experiment because the existing theories of electromagnetics differ widely in their views as to the distribution of the force in the neighbourhood of an unclosed current. We therefore place the plane of our circle in a vertical position, bring the spark-gap to the highest point, and by turning the circle about a vertical axis we try to ascertain in what position the sparks are longest and in what position they disappear or nearly disappear. But when we get to a distance of 1-1·5 metre from the primary oscillation, are met by an unexpected difficulty. For the maxima and nima lose their distinctness, except in particular positions, so at it becomes difficult to adjust the position of the circle for either; indeed, at certain places the differences between the spark-lengths during a revolution of the circle are so small that it becomes impossible to specify any definite direction of the force. Now observe that this difficulty again disappears when we pass beyond a distance of about 2 metres. Certainly the sparks are now very small and need to be observed in the dark and with the aid of a lens; but they disappear sharply in a definite position of the circle, and are strongest in a position at right angles to this. When the distance is further increased the spark-length only diminishes slowly. I have not been able to decide the farthest distance at which they could be observed. When I placed the primary conductor in one corner of a large lecture-room 14 metres long and 12 metres broad, the sparks could be perceived in the farthest parts of the room; the whole room seemed filled with the oscillations of the electric force. It is true that in the neighbouring rooms the action could not be perceived even at small distances; solid walls exercise a powerful damping effect upon it. In the lecture-room referred to I ascertained the distribution of the force as follows:—Wherever the direction of the force could be definitely determined I marked it by a chalk line on the floor; but wherever it could not well be determined I drew a star upon the floor. Fig. 23 shows on a reduced scale a portion of the diagram thus made; with reference to it we note:—

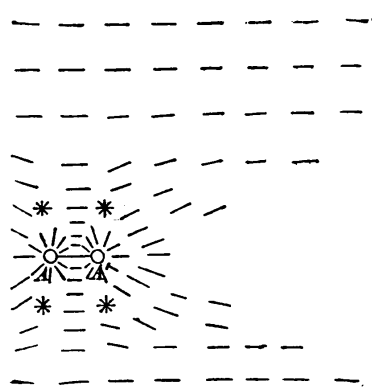

Fig. 23.

1. At distances beyond 3 metres the force is everywhere parallel to the primary oscillation. This is clearly the region in which the electrostatic force has become negligible, and the electromagnetic force alone is effective. All theories agree in this—that the electromagnetic force of a current-element is inversely proportional to the distance, whereas the electrostatic force (as the difference between the effects of the two poles) is inversely proportional to the third power of the distance.

It is worthy of notice that, in the direction of the oscillation, the action becomes weaker much more rapidly than in the perpendicular direction, so that in the former direction the effect can scarcely be perceived at a distance of 4 metres, whereas in the latter direction it extends at any rate farther than 12 metres. Many of the elementary laws of induction which are accepted as possible will have to be abandoned if tested by their accordance with the results of these experiments.

2. As already stated, at distances less than a metre the character of the distribution is determined by the electrostatic force.

3. Along one pair of straight lines the direction of the force can be determined at every point. The first of these straight lines is the direction of the primary oscillation itself; the second is perpendicular to the primary oscillation through its centre. Along the latter the magnitude of the force is at no point zero; the size of the sparks induced by it diminishes steadily from greater to smaller values. In this respect also the phenomena contradict certain of the possible elementary laws which require that the force should vanish at a certain distance.

4. One remarkable fact that results from the experiment is, that there exist regions in which the direction of the force cannot be determined; in our diagram each of these is indicated by a star. These regions form in space two rings around the rectilinear oscillation. The force here is of approximately the same strength in all directions, and yet it cannot act simultaneously in these different directions; hence it must assume in succession these different directions. Hence the phenomenon can scarcely be explained otherwise than as follows:—The force does not retain the same direction and alter its magnitude; its magnitude remains approximately constant, while its direction changes, passing during each oscillation round all the points of the compass. I have not succeeded in finding an explanation of this behaviour, either in the terms which have been neglected in our simplified theory, or in the harmonics which are very possibly mingled with our fundamental vibration. And it seems to me that none of the theories which are based upon the supposition of direct action-

at-a-distance would lead us to expect anything of this kind. But the phenomenon is easily explained if we admit that the electrostatic force and the electromagnetic force are propagated with different velocities. For in the regions referred to these two forces are perpendicular to one another, and are of the same order of magnitude; hence if an appreciable difference of phase has arisen between them during the course of their journey, their resultant—the total force—will, during each oscillation, move round all points of the compass without approaching zero in any position.

A difference between the rates of propagation of the electrostatic and electromagnetic forces implies a finite rate of propagation for at least one of them. Thus it seems to me that we probably have before us here the first indication of a finite rate of propagation of electrical actions.

In an earlier paper[1] I mentioned that trivial details, without any apparent reason, often interfered with the production of oscillations by the primary spark. One of these, at any rate, I have succeeded in tracing to its source. For I find that when the primary spark is illuminated, it loses its power of exciting rapid electric disturbances. Thus, if we watch the sparks induced in a secondary conductor, or in any auxiliary conductor attached to the discharging circuit, we see that these sparks vanish as soon as a piece of magnesium wire is lit, or an arc light started, in the neighbourhood of the primary spark. At the same time the primary spark loses its crackling sound. The spark is particularly sensitive to the light from a second discharge. Thus the oscillations always cease if we draw sparks from the opposing faces of the knobs by means of a small insulated conductor; and this even though these sparks may not be visible. In fact, if we only bring a fine point near the spark, or touch any part of the inner surfaces of the knobs with a rod of sealing-wax or glass, or a slip of mica, the nature of the spark is changed, and the oscillations cease. Some experiments made on this matter seem to me to prove (and further experiments will doubtless confirm this) that in these latter cases as well the effective

[1] See No. II., p. 29.

cause of the change is the light of a side-flash, which is scarcely visible to the eye.

These phenomena are clearly a special form of that action of light upon the electric discharge, of which one form was first described by myself some time ago, and which has since been studied in other forms by Herren E. Wiedemann, H. Ebert, and W. Hallwachs.